A LOGIC OF SEQUENCES

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ABSTRACT

The notion of “sequences” is fundamental to practical reasoning in computer science, because it can appropriately represent “data (information) sequences”, “program (execution) sequences”, “action sequences”, “time sequences”, “trees”, “orders” etc. The aim of this paper is thus to provide a basic logic for reasoning with sequences. A propositional modal logic LS of sequences is introduced as a Gentzen-type sequent calculus by extending Gentzen’s LK for classical propositional logic. The completeness theorem with respect to a sequence-indexed semantics for LS is proved, and the cut-elimination theorem for LS is shown. Moreover, a first-order modal logic FLS of sequences, which is a first-order extension of LS, is introduced. The completeness theorem with respect to a first-order sequence-indexed semantics for FLS is proved, and the cut-elimination theorem for FLS is shown. LS and the monadic fragment of FLS are shown to be decidable.

Keywords: sequence, computer science, gentzen-type sequent calculus, fls
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A b s t r a c t. The notion of “sequences” is fundamental to practical reasoning in computer science, because it can appropriately represent “data (information) sequences”, “program (execution) sequences”, “action sequences”, “time sequences”, “trees”, “orders” etc. The aim of this paper is thus to provide a basic logic for reasoning with sequences. A propositional modal logic LS of sequences is introduced as a Gentzen-type sequent calculus by extending Gentzen’s LK for classical propositional logic. The completeness theorem with respect to a sequence-indexed semantics for LS is proved, and the cut-elimination theorem for LS is shown. Moreover, a first-order modal logic FLS of sequences, which is a first-order extension of LS, is introduced. The completeness theorem with respect to a first-order sequence-indexed semantics for FLS is proved, and the cut-elimination theorem for FLS is shown. LS and the monadic fragment of FLS are shown to be decidable.
1. Introduction

The aim of this paper is to provide a basic logic for reasoning with sequences. The notion of “sequences” is fundamental to practical reasoning in computer science, because it can appropriately represent “data (information) sequences”, “program (execution) sequences”, “action sequences”, “time sequences”, “word (character or alphabet) sequences”, “DNA sequences” etc. The notion of sequences is thus useful to represent the notions of “information”, “computation”, “trees”, “orders”, “preferences”, “strings”, “vectors” and “ontologies”. In a view of reasoning with sequences, dynamic logics [6] are logics dealing with program (or action) sequences, temporal logics [5] are logics dealing with time sequences, and Lambek calculus [11] is a logic dealing with word sequences. Representing “information” by sequences is particularly suitable and important, since a sequence structure gives a monoid $\langle M, ;, \emptyset \rangle$ with an informational interpretation [16]:

1. $M$ is a set of pieces of (ordered or prioritized) information (i.e., a set of sequences),
2. $;$ is a binary operator (on $M$) which combines two pieces of information (i.e., the concatenation operator on sequences),
3. $\emptyset$ is the empty piece of information (i.e., the empty sequence).

The informational interpretation for a monoid based semantics for substructural logics including Lambek calculus was proposed by Wansing [16] extending and generalizing Urquhart’s interpretation [14] for semilattice (i.e., idempotent commutative monoid) semantics for relevant logics.

Handling the notion of sequences in a logic has recently been studied by several researchers. Sequence logic (SL), which is a parameterized logic where the formulas are sequences of formulas, was proposed and studied by Walicki et al. [15, 1]. In [1], the completeness and decidability theorems w.r.t. the class of dense linear orderings and the class of linear orderings are proved for SL. A predicate logic with sequence variables and sequence function symbols was introduced by Kutsita and Buchberger [10]. In [10], a Gentzen-type sequent calculus $G^\cong$ for this logic was introduced, and the completeness theorem for $G^\cong$ was proved. The three approaches based on